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SENSING STOCHASTICITY OF ATOMIC SYSTEMS IN CROSSED ELECTRIC AND MAGNETIC FIELDS BY ANALYSIS OF LEVEL STATISTICS FOR CONTINUOUS ENERGY SPECTRA

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Abstract

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A new method for sensing stochasticity and chaotic elements in dynamics of atomic and nano-optical systems in the crossed external electric and magnetic fields is developed. It is based on the quantum approach to calculation of the energy levels spectra and analysis of the level statistics for continuum are used. Some illustrations regarding the stochasticity and quantum chaos in non-H atomic system (*Li*) are presented.

Key words: sensing stochasticity, atomic system, energy levels statistics, electric and magnetic fields

Анотація

ДЕТЕКТУВАННЯ СТОХАСТИЧНОСТІ АТОМНИХ СИСТЕМ У СХРЕЩЕНИХ ЕЛЕКТРИЧНОМУ І МАГНІТНОМУ ПОЛЯХ НА ПІДСТАВІ АНАЛІЗУ СТАТИСТИКИ РІВНІВ У НЕПЕРЕРВНОМУ ЕНЕРГЕТИЧНОМУ СПЕКТРІ

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Розвинуто новий підхід до детектування стохастичності і елементів хаосу в динаміці атомних і нано-оптичних систем у схрещених електричному та магнітному полях, який базується на квантовому методі розрахунку спектру енергетичних рівнів та аналізі статистики рівнів у континуумі. Наведені чисельні ілюстрації прояву стохастичності та квантового хаосу в неводневій атомній системі.

Ключові слова: детектування стохастичності, атомна система, статистика рівнів енергії, електричне і магнітне поле

Аннотация**ДЕТЕКТИРОВАНИЕ СТОХАСТИЧНОСТИ АТОМНЫХ СИСТЕМ В СКРЕЩЕННЫХ ЭЛЕКТРИЧЕСКОМ И МАГНИТНОМ ПОЛЯХ НА ОСНОВЕ АНАЛИЗА СТАТИСТИКИ УРОВНЕЙ В НЕПРЕРЫВНОМ ЭНЕРГЕТИЧЕСКОМ СПЕКТРЕ***С. В. Амбросов, В. М. Игнатенко, Д. А. Корчевский, В. П. Козловская*

Развит новый подход к детектированию стохастичности и элементов хаоса в динамике атомных и нано-оптических систем в скрещенных электрическом и магнитном поле, базирующийся на квантовом методе расчета спектра энергетических уровней и анализе статистики уровней в континууме. Приведены численные иллюстрации проявления стохастичности и квантового хаоса в неводородной атомной системе.

Ключевые слова: детектирование стохастичности, атомная система, статистика уровней энергии, электрическое и магнитное поле

1. Introduction

In last years a great interest attracts studying a dynamics of quantum systems in external electric and magnetic field [1-21]. It is provided by importance of studying a phenomenon of stochasticity or quantum chaos in atomic systems for further understanding manifestation of the chaotic features in a work of different electronic devices and systems, including the nano-optical ones.

One of the most important discoveries of last decades is the point that dynamics of these systems in external electromagnetic fields has features of the random, stochastic kind and its realization does not require the specific conditions. One should mention an effect of crossing the energy levels of atomic systems in the external electric and magnetic fields. It can be treated as a real manifestation of quantum chaos. This effect is the physical basis for sensing very weak magnetic field strength. [1-9]. It is important to note that the same phenomena may occur in the exciton dynamics in semiconductors as an exciton is similar in many features to the hydrogen atom, which has been considered as a prototype of the quantally chaotic system [9].

The Stark and Zeeman effects are separately considered in a great number of papers, but a behaviour of hydrogen and non-hydrogen atomic systems, the Wannier-Mott excitons in crossed electric and magnetic field is studied significantly less. Rydberg atoms in strong external fields have been shown to be real physical examples of non-integrable systems for studying the quantum manifestations of classical chaos both experimentally and theoretically [3-11]. To describe these phenomena, one has to make calculating and interpreting the recurrence spectra which is the Fourier transformation of a photo ab-

sorption spectrum [16-19]. The recurrence spectrum provides a quantum picture of classical behaviour. Studies of recurrence spectra have led to observations of the creation of new orbits through bifurcation, the onset of irregular behaviour through core scattering and symmetry breaking in crossed fields [2-6,10,14-19]. In the past, many researchers have calculated the recurrence spectra of a Rydberg atom in an external field. But they only calculated the spectra in static electric or magnetic fields. In a recent experiment, the absorption spectrum of the lithium atom in a static electric field plus a weak oscillating field was measured and Haggerty and Delos gave some explanation for it theoretically (c. f. [2-4,16-18]). But as to the influence of an oscillating electric field on the absorption spectrum of the Rydberg atom in static magnetic field, none has given the calculation both experimentally and theoretically, besides the first classical estimate [18].

In ref. [19] a new scheme for sensing stochasticity and chaotic features of atomic (nano-optical) systems in the crossed external electric and magnetic fields has been developed and based on the new quantum approach to calculating the recurrence energy spectra for atomic systems in crossed fields in chaotic regime [20,21] (c. f. [10,11]) and some experimental data for checking obtained results. This paper goes on our investigations and is devoted to development of a new method for sensing stochasticity and chaotic elements in dynamics of atomic and nano-optical systems in the crossed external electric and magnetic fields. As a basis for developing a new method for sensing stochasticity we use new quantum approach to calculation of the energy levels spectra and an analysis of the level statistics for continuum. Some illustrations regarding mani-

festation of the stochasticity and quantum chaos elements in a number of atomic systems are presented.

2. An atomic system in crossed electric and magnetic fields: Calculation of energy levels spectra and analysis of level statistics

To calculate the energy spectra of atomic system in the crossed external electric and magnetic fields we use earlier developed approach, based on solution of the 2-dimensional Schrödinger equation [20,21] for an atomic system in crossed fields and operator perturbation theory [10]. For definiteness, we consider a dynamics of the complex non-coulomb atomic systems in a static magnetic and electric fields. The hamiltonian of the multi-electron atom in a static magnetic and electric fields is (in atomic units) as follows:

$$H = 1/2(p_\rho^2 + l_z^2 / \rho^2) + Bl_z / 2 + (1/8)B^2 \rho^2 + (1/2)p_z^2 + Fz + V(r) \quad (1)$$

where the electric field F and magnetic field B are taken along the z -axis in a cylindrical system; In atomic units: 1 a. u. $B=2.35 \cdot 10^5$ T, 1a. u. $F=5,144 \cdot 10^6$ kV/cm. If one consider only the $m=0$ state, thus $l_z=0$; $V(r)$ is a one-electron model potential, which is chosen in the following form:

$$V(r) = -1/r + V_{el}, \quad (2a)$$

$$V_{el} = V_1 + V_2 + V_3$$

Here $V_i (i=1,2,..)$ is a potentials of the K-shell (as in a case of the Li atom) and other atomic shells. This potential is usually defined as:

$$V_1(r) = (2/Zr)[1 - \exp(-2b_1 r)(1+r)], \quad (2b)$$

Here Z is a nuclear charge, and b is free length parameter, which is chosen to give the energy spectrum of free atom. For solution of the Schrödinger equation with hamiltonian equations (1,2) we constructed the finite differences scheme which is in some aspects similar to method [13]. An infinite region is exchanged by a rectangular region: $0 < \rho < L_\rho$, $0 < z < L_z$. It has sufficiently large size; inside it a rectangular uniform grid with steps h_ρ , h_z was constructed. The external boundary condition, as usually, is: $(\partial\Psi/\partial n)_r = 0$. The knowledge of the asymptotic behaviour of wave function in the infinity allows to get numeral estimates for L_ρ , L_z . A wave function has an asymptotic of the kind as: $\exp[-(-2E)^{1/2}r]$, where $(-E)$ is the ionization energy

from stationary state to lowest Landau level. Then L can be estimated as $L \sim 9(-2E)^{-1/2}$. The more exact estimate is found empirically. The difference scheme is constructed as follows. The three-point symmetric differences scheme is used for second derivative on z . The derivatives on ρ are approximated by $(2m+1)$ -point symmetric differences scheme with the use of the Lagrange interpolation formula differentiation. The eigen-values of hamiltonian are calculated by means of the inverse iterations method. The corresponding system of inhomogeneous equations is solved by the Thomas method. To increase an accuracy of the calculated eigen values, the Richardson extrapolation method on the grid step is used (c. f. [13,21]). To calculate the values of the width G for resonances in spectra of atomic system in crossed electric and magnetic field one can use the modified operator perturbation theory method (see details in ref. [10,20]). Note that the imaginary part of the state energy in the lowest PT order is defined as follows:

$$\text{Im} E = G/2 = \pi \langle \Psi_{Eb} | H | \Psi_{Es} \rangle^2 \quad (3)$$

with the total Hamiltonian of system in an electric and magnetic field. The state functions Ψ_{Eb} and Ψ_{Es} are assumed to be normalized to unity and by the $\delta(k - k')$ -condition, accordingly. Other calculation details can be found in ref. [19-21]. Further it is convenient to introduce new "field" parameters [14]:

$$c_B = (1/8)\eta^4 B^2, \quad c_F = \eta^3 F \quad (4)$$

where $\eta = 1/(-2E)^{1/2}$. Let us define the energy level density $\rho(\eta)$ as:

$$\rho(\eta) = \{ \text{No. of eigen-values in } [\eta - \Delta/2, \eta + \Delta/2] \} / \Delta \quad (5)$$

where Δ is some interval small, but still sufficiently greater than mean level spacings. The main task is to get statistical information about continuous energy spectrum and data about statistics of nearest-neighbour spacings of energy levels (NNS). According to ref. [14], one first has to rectify the discrete spectra that can be done by usual way. One has to introduce the counting function $N(\eta)$, which counts the number of eigen-values less than η :

$$N(\eta) = \sum_i \Theta(\eta - \eta_i) \quad (6)$$

and is interpolated by a smooth function $\bar{N}(\eta)$. Further one can get rid of the global behaviour by choosing as new variables e_i instead of η_i with $e_i = \bar{N}(\eta_i)$. For the resulting spectrum of the e_i values, the average number of levels per unit interval is

equal to 1. The NNS s_i are defined as $s_i = e_{i+1} - e_i$ and their distribution $p(s)$ is defined as follows:

$$\text{Prob. (some } s_i \text{ in } [s, s+\Delta s]) = \int_s^{s+\Delta s} p(s) ds \quad (7)$$

As $p(s)$ is defined to be normalized and the average number of levels per unit interval is 1, the distribution

$$p(s) \text{ has mean 1, i. e. } \int_0^{\infty} sp(s) ds = 1$$

3. Results and discussion

Let us give now an analysis of the sensing specific quantum chaotic features in the atomic systems in the crossed external electric and magnetic fields. We have used our approach to calculation of the energy levels spectra for hydrogen and lithium atom in a crossed electric and magnetic fields and have obtained information about continuous energy

spectrum and the NNS statistics. We have used in calculations of hydrogen and lithium atoms the field parameters (4) in the range 0,1-2,0 and basis dimensions $D=4 \cdot 10^3$. The hydrogen atom has been used a test in order to compare our results with analogous data of ref. [14]. The details analysis has shown that our results and data from ref. [14] are in very close agreement. Though there is some numerical difference, but the key result, connected with realization of the Wigner and Poisson distributions for $p(s)$, is reached within two different quantum methods. Further we have calculated the level density $\rho(\eta)$ for the lithium atom. Figure 1 shows the behaviour of eigen-value density for various values of the field parameters c_B and c_F . Increasing c_B (under fixed value of c_F) has the main effect of moving the maximum to higher η values (see fig. 1a). But, an increasing c_F (under fixed value of c_B) has the main effect of moving maximum to lower η values (fig. 1b).

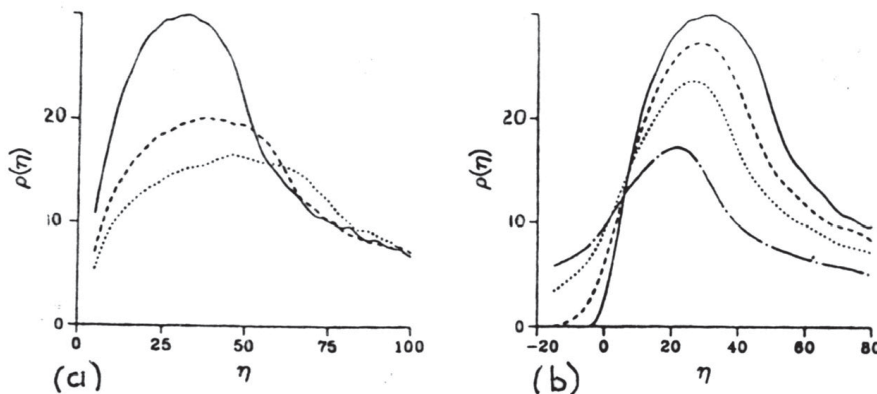


Fig. 1. Eigen-value density for various values of the field parameters c_B and c_F . (Li): (a) $D_N=3,6 \cdot 10^3$; $c_F=0,1$; $c_B=0,1$ (—); $c_B=0,5$ (---); $c_B=0,9$ (.....); (b) $D_N=3,6 \cdot 10^3$; $c_B=0,1$; $c_F=0,1$ (—); $c_F=0,5$ (---); $c_F=0,9$ (.....); $c_F=1,8$ (-.-);

This is in a full analogy with the hydrogen case. We have calculated the nearest-neighbour distribution $p(s)$ for different values of the field parameters and D_N . Figure 2 shows a dependence of the $p(s)$ upon D_N for values of $c_F=0,1$; $c_B=0,9$ and $c_F=0,9$; $c_B=0,1$. The main result is that all curves are corresponding fits to Brody distribution: $p_q(s) = A s^q \exp(-\alpha s^{q+1})$, where $\alpha = [\Gamma\{(q+2)/(q+1)\}]^{q+1}$ and $A = (q+1)\alpha$. For $q=0$ and $q=1$ the Brody distribution becomes the Poisson $p_p(s) = \exp(-s)$ and Wigner $p_w(s) = 0,5\pi \exp(-0,5\pi s^2)$ distribution respectively. The distribution in fig 2a (parameters $c_B=0,9$; $c_F=0,1$) is very close to a Poisson distribution. At the same time distribution in fig 2b (the field parameters

$c_B=0,1$; $c_F=0,9$) is practically a Wigner distribution. This case is corresponding to realization of the quantum chaos phenomenon.

Let us remember that this phenomenon is connected with availability of multiple resonances in atomic spectra in the crossed external fields and provided by their interference phenomena and quantum fluctuations, which characterize the chaotic system [1]. So, we presented an effective scheme for sensing stochasticity and chaotic elements in dynamics of atomic systems in the crossed external electric and magnetic fields, which is based on our new quantum approach to calculation of the energy levels spectra and analysis of the level statistics for continuum.

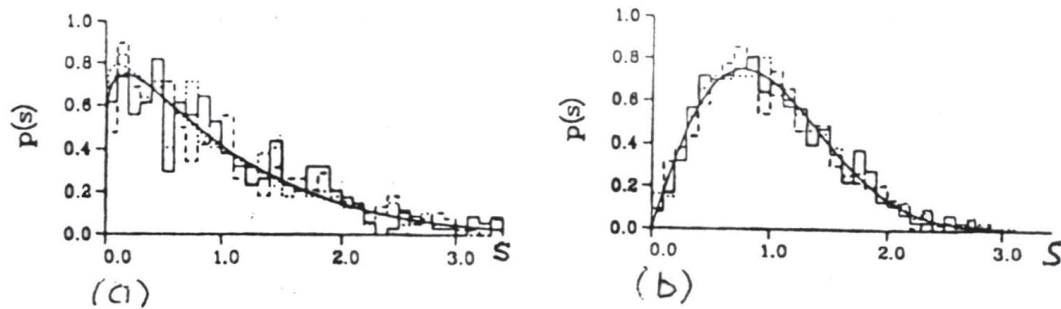


Fig. 2. Convergence of NNS distribution for increasing D_N and field parameters values (Li): (a) $c_B=0,9$; $c_F=0,1$; $D_N=2,4\cdot 10^3$ (____); $3,6\cdot 10^3$ (.....); $4,8\cdot 10^3$ (—); $p_p(s)$ (b) $c_B=0,1$; $c_F=0,9$; $D_N=2,4\cdot 10^3$ (____); $3,6\cdot 10^3$ (.....); $4,8\cdot 10^3$ (—); $p_w(s)$

References

- Gutzwiller M., Chaos in Classical and Quantum Mechanics. — N. — Y.: Springer, 1990. — 720p.
- Kleppner D., Chun-Ho I., Welch G. R., Irregular Atomic Systems and Quantum Chaos, Ed. J. C. Gay, N-Y.: Kluwer, 1990. — P. 21-48.
- Dupret K., Zakrzewski J., Delande D., Resonances in the Diamagnetic Rydberg Spectrum: Order and Chaos// Europhys. Lett. — 1995. — V. 31(5). — P. 251-256.
- Gasati G., Guarneri I., Mantica G., Classical Stabilization of periodically kicked hydrogen atoms// Phys. Rev. A. — 1994. — V. 50(6). — P. 5018-5024.
- Dando P. A., Monteiro T. S. Atoms in Static Fields: Chaos or Diffraction// In: Photonic, Electro-nic, Atomic Collisions, Eds. W. Aumyr, H. Winter. — World Sci., Singapore. — 1997. — P. 621-630
- Benvenuto F., Casati G., Shepelyansky D. L., Rydberg Stabilization of atoms in strong fields: "magic" mountain in chaotic sea// Z. Phys. B. — 1994. — V. 94. — P. 481-486.
- Delone N. B., Krynov V. P. Atom in a strong light field. — M.: Atomizdat, 1994. — 370p.
- Landau L. D., Lifshits E. M. Quantum Mechanics. — M.: Nauka, 1989.
- Lisitsa V. S., New in Stark and Zeemane effects for hydrogen atom// Usp. Phys. Nauk.. — 1987. — V. 153. — P. 379-422.
- Glushkov A. V., Ivanov L. N., DC Strong-Field Stark-Effect: consistent quantum-mechanical approach// J. Phys. B:At. Mol. Opt. Phys. — 1993. — V. 26,N16. — P. L379-388.
- Glushkov A. V., Ambrosov S. V. et al, Resonances in Quantum Systems in strong external fields: Consistent Quantum Approach// Journ. Techn. Phys. — 1997. — V. 37(2). — P. 215-218.
- Popov V. S., Karnakov B. M., Moor V. D., Ionization of atoms in electric and magnetic field and methos of imaginary time// JETP. — 1998. — V. 113(5). — P. 1579-1605.
- Ivanov M. V., Schnelcher P., Ground States of H, He, ..., Ne and their singly positive ions in strong magnetic fields: the high-field regime// Phys. Rev. A. — 2000. — V. 61. — P. 2251-2263.
- Hegerfeldt G. C., Henneberg R., Level statistics for continuous energy spectra with application to H atom in crossed electric and magnetic field// Phys. Rev. A. — 1994. — Vol. 49. — P. 3531-3540.
- Fielding H., Wals. J., van der Zande W., van den Heuvel H., Rydberg-e wave-packet dynamics in electromagnetic fields and evidence for stabilization// Phys. Rev. A-1995-Vol. 51. — P. 611-618.
- Cheng T., Liu J., Chen S., Guo H. Rydberg atoms in parallel microwave and magnetic fields- classical dynamics// Phys. Lett. A. — 2000. — V. 265. — P. 384-390.
- Rao J., Delande D., Taylor K., Quantum manifestations of scattering orbits in scaled spectrum of non-hydrogenic atom in crossed fields// J. Phys. B:At. Mol. Opt. Phys. — 2002. — V. 35. — P. L1-10.
- Wang D., Din S., Liu S. Influence of oscillating electric field on recurrence spectra of Li Rydberg atom in strong magnetic field// J. Phys. B:At. Mol. Opt. Phys. — 2003. — V. 36-P. 4225-4232.
- Korchevsky D. A., Shpinareva I. M., Ignatenko A. V., Sensing stochasticity of atomic systems by analysis of recurrence spectra in an crossed dc magnetic and ac electric fields // Sensor Electr. & Microsyst. Techn. — 2005. — N1. — P. 21-26.
- Glushkov A. V., Ambrosov S. V., Ignatenko A. V., Korchevsky D. A., DC Strong Field Stark Effect for Non-hydrogenic Atoms: Consistent Quantum Mechanical Approach // Int. Journ. Quant. Chem. — 2004. — Vol. 99,N5. — P. 936-939.
- Korchevsky D. A. Energy spectra of Wannier-Mott excitons and hydrogen atom in an crossed dc electric and magnetic fields// Photoelectronics. — 2003. — V. 13. — P. 131-136.