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CHARACTERISTIC PROPERTIES OF OPTO-ACOUSTIC INTERACTION IN THE "THICK" ACOUSTIC GRATING

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Abstract. The theoretical analysis of the diffraction spectrum at the normal incidence of the plane light wave onto a sound wave in the isotropic medium is developed. In the framework of the bound waves pattern the diffraction spectrum behavior is investigated. At the same time the parameters of the sound wave such as a width and intensity of the sound beam was modifying. As a result of this investigation it was showed that the behavior of the diffraction maximums intensity depending on intensity of the sound intensity is essentially modified under increase of the width of the acousto-optic layer even at the orthogonal orientation of interacting fields

Keywords: acousto-optic effect, Raman-Nath diffraction, Bragg diffraction

ОСОБЛИВОСТІ АКУСТООПТИЧНОЇ ВЗАЄМОДІЇ В "ТОВСТІЙ" АКУСТИЧНІЙ ГРАТЦІ

Л. В. Михайловська, А. С. Михайловська

Анотація. Проведено теоретичний аналіз дифракційного спектру у випадку ортогонального падіння плоскої світової хвилі на звукову хвилю в ізотропному середовищу. При цьому в рамках моделі зв'язаних хвиль досліджується поведінка дифракційного спектру при зміненні параметрів звукової хвилі, зокрема, ширини та інтенсивності звукової хвилі. Показано, що при збільшенні товщини прошарку акустооптичної взаємодії поведінка інтенсивності світла в дифракційних максимумах в залежності від інтенсивності звука суттєво змінюється навіть при ортогональній орієнтації взаємодіючих полів.

Ключові слова: акустооптичний ефект, дифракція Рамана-Ната, дифракція Брега

ОСОБЕННОСТИ АКУСТООПТИЧЕСКОГО ВЗАИМОДЕЙСТВИЯ В “ТОЛСТОЙ” АКУСТИЧЕСКОЙ РЕШЕТКЕ

Л. В. Михайловская, А. С. Михайловская

Аннотация. Проведен теоретический анализ дифракционного спектра в случае ортогонального падения плоской световой волны на звуковую волну в изотропной среде. При этом в рамках модели связанных волн исследуется поведение дифракционного спектра при изменении параметров звуковой волны, в частности, ширины и интенсивности звукового пучка. Показано, что при увеличении ширины слоя акустооптического взаимодействия поведение интенсивности света в дифракционных максимумах в зависимости от интенсивности звука существенно меняется даже при ортогональной ориентации взаимодействующих полей.

Ключевые слова: акустооптический эффект, дифракция Рамана-Ната, дифракция Брэгга

Introduction

The interest to investigations of opto-acoustic interaction is determined by extensive practice application of the acousto-optic methods for effective control of the space-temporary parameters of an optic radiation [1-3], for laser diagnostic of acoustic fields in liquids and gases, in the area of the measuring of the moving of liquids and gases [4,5]. In the latter years the works with using of the acousto-optic interaction for investigation optic characteristic of the scattering mediums, in particular, for acousto-optic visualization in turbid mediums, were published [5-7]. The interest to this works is stimulated by modern applications of optic methods for the medical diagnostic [7].

In the many tasks which are connected with light diffraction by sound wave it is necessary to know as those or other sound characteristics have an effect upon characteristics of passed light beam.

It is well known that the light diffraction by sound waves depends on incidence angle of light, on wave length of the incidence light, on length of the sound wave, on intensity and width of the sound beam. However at the theoretical analysis of this problem the most authors are guided themselves by approximations of the limit cases, so in the result the applications of finding results have limit. In the present wok the intermediate case of diffraction between the Raman-Nath’s regime and Bragg’s diffraction is discussed greater detail

Theoretical analysis

In many papers the theory of the acousto-optics interaction is built upon base of general solutions of the wave equations obtained from Maxwell’s equations [1,8-11].

Let in an isotropic medium the electromagnetic wave $E = E_0 \exp\left[j\left(k_{0y} y + k_{0z} z - \omega_0 t\right)\right]$ falls on the plane $z = 0$ at the angle θ_0 to axis z. A plane acoustic wave propagates along y axis between the planes $z = 0$ and $z = L$. For presenting geometry on account of the symmetry of our task it could consider that all fields are not depend on x coordinate. In the case of using an approximation of plane acoustic and optic waves the wave equation in the region of the interaction of light and sound ($0 \le z \le L$) can be write in a form $\frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\varepsilon \cdot E)$. Here perturbed by sound the dielectric permeability of the medium has form $\varepsilon(y) = n^2 \approx n_0^2 - 2n_0 \Delta n_0 \cos(k_s y - \Omega t)$, where n_0 — the refractive index in the absence of sound, $\Delta n_0 = \sqrt{0.5 \cdot M \cdot I_s}$ — amplitude of modulation of the refractive index by sound wave, M — acousto-optic quality factor, I_s — intensity of sound wave, $k_s = \frac{2\pi}{\Lambda}$, Λ, Ω — wave number, length and frequency of sound wave respectively. It is clear, that the perturbations of the refractive index and dielectric permeability by sound wave considerably depend on the sound intensity.

If angle of incidence light beam $\theta_0 \ll 1$ and $\lambda \ll \Lambda$ (or $k_s \ll k$), then solutions of given wave equation may be search in the form of the expansion in series of the plane waves with slowly verified amplitudes [14]

$$E(y,z,t) = \exp\left[j\left(k \sin\theta_0 \cdot y + \right.\right.$$
$$\left.\left. + k\cos\theta_0 \cdot z - \omega_0 t\right)\right] \sum_{-\infty}^{\infty} V_m(z) \exp\left[jm\left(k_s y - \Omega t\right)\right]. \quad (1)$$

In accordance with choice of such series expansion the incident light beam breaks up into series of plane waves. These waves propagate at small angles θ_m relative to the direction of the incident light beam. The following relations define values of these angles $\sin\theta_m = \sin\theta_0 + m\frac{k_s}{k} = \sin\theta_0 + m\frac{\lambda_0}{n_0\Lambda}$. Here $m = 0, \pm 1, \pm 2, ...$ — diffraction orders.

Substitution of the expansion (1) into wave equation makes it possible to derive differential equations for finding of the amplitudes $V_m(z)$. These equations have form of the recurrence relations. Using the approximation of the relatively slow changes of the functions $V_m(z)$ in the range $0 \le z \le L$ the following infinite system of the first order differential equations was obtained for determination $V_m(z)$ [1,13,14]

$$\frac{dV_m}{dz} + j\mu_m V_m = j\frac{q}{2}\left(V_{m+1} + V_{m-1}\right). \qquad (2)$$

Here $\mu_m = \frac{mk_s\left(2k\sin\theta_0 + mk_s\right)}{2k\cos\theta_0} =$

$$= \frac{2\pi m}{\Lambda\cos\theta_0}\left(\sin\theta_0 + m\frac{\lambda_0}{2n_0\Lambda}\right),$$

$$q = \frac{k\Delta n_0}{n_0\cos\theta_0} = \frac{2\pi\Delta n_0}{\lambda_0\cos\theta_0}.$$

This system of equations (2) must be solved with boundary conditions $V_0(0) = E_0$ and $V_m(0) = 0$ for all $m \ne 0$. The relation $I_m = V_m \cdot V_m^*$ determines intensity of light in m diffraction maximum. Number of the excite diffraction maximums depends on both intensity of sound wave and width of sound beam.

In present work the case of orthogonal incidence of plane light wave onto the sound wave in the isotropic medium are considered. At the same time in the framework of the stated above model of the coupled waves the dependence of diffraction spectrum on the parameters of sound wave namely width and intensity of sound wave is investigated.

In the case of perpendicular incidence of light ray onto the sound beam the parameters μ_m, μ_{-m} take the following forms $\mu_m = \mu_{-m} = m^2\frac{\pi\lambda_0}{n_0\Lambda^2}$. The energy of the incidence radiation disperses between set of the diffraction orders symmetrically relative to transmitted light, i.e. (that is) $V_m = V_{-m}$. The system of equations (2) becomes simpler and takes on form

$$V_0' = jqV_1, \; V_m' + j\mu_m V_m =$$
$$= 0.5jq\left(V_{m-1} + V_{m+1}\right), \; m = 1,2,3... \qquad (2a)$$

The intensity of incident radiation is defined by expression $I_i = E_0^2 = I_0(z) + 2\sum_1^m I_m(z)$.

Under the condition $Q = \frac{k_s^2}{k}L = \frac{2\pi\lambda_0}{\Lambda^2 n_0}L \ll 2\pi$ the Raman-Nath diffraction mode takes place. For this mode the approximation of the two-dimensional (plane) phase grating is true. For this approximation the diffraction maximum distribution of the light intensity at going out from sound layer is described by Bessell functions $I_{mB} = I_i \cdot J_m^2(qL)$. Here $qL = \frac{2\pi\Delta n_0}{\lambda_0}L$ — dimensionless Raman-Nath parameter. (It should be noted that Raman-Nath approximation follow from (2) if all parameters $\mu_m = 0$). In accordance with these expressions an increase of the modulation amplitude of the refractive index Δn_0 (that is sound intensity) affects on diffraction phenomena as well as an increase of the sound field width L. So for Raman-Nath approximation number of the excited diffraction maximums by same way depends on sound intensity and width of sound beam. This number may be estimated by using relation $J_0^2(qL) + 2\sum_1^m J_m^2(qL) = 1$. It is easily to make sure by direct calculation that equality $J_0^2(qL) + 2\sum_1^m J_m^2(qL) \simeq 1$ is satisfied sufficiently exactly if $qL \le m$. For example, for $qL = 3$ it is easy to calculate sum $J_0^2 + 2\cdot\left(J_1^2 + J_2^2 + J_3^2\right) = 0.9613$. So it may be expect that the number of the observable diffraction maximums in each concrete case do not exceed considerably the magnitude $m = qL$.

In present paper the condition of smallness of the sound beam width that necessary to satisfy condition $Q \ll 2\pi$ does not use. The system of equations (2a) was solved successively for three cases: 1) $m = 0, 1$. 2) $m = 0, 1, 2$. 3) $m = 0, 1, 2, 3$. Thereby we neglect by diffraction in more high orders 1)second, 2)third, 3)fourth respectively. We try to estimate accuracy our used approximations by successively increasing of the number the diffraction maximums that taken into account in numerical calculations. So, if in some of region of system parameters the accounting $m+1$ diffraction orders do not vary but only define more exactly solutions for m diffraction orders then it can state that for given region it may be confined oneself only by m diffraction orders.

In the beginning we consider first case of diffraction permitting analytic solution. In this case after sound beam passage by light beam only two symmetrical maximums relative to the transmitted main light beam are observed. This means that it is need to solve next system of two differential equations

$$\begin{cases} V_0' = jqV_1 \\ V_1' + j\mu_1 V_1 = 0.5jqV_0. \end{cases} \tag{3}$$

With accounting of boundary conditions $V_0(0)=E_0$, $V_1(0)=0$ the following solutions for amplitudes of the diffraction maximums were obtained

$$V_0(z)=\frac{E_0}{r_2-r_1}\left(r_2 e^{jr_1 z}-r_1 e^{jr_2 z}\right),$$
$$V_1(z)=\frac{E_0 r_1 r_2}{q(r_2-r_1)}\left(e^{jr_1 z}-e^{jr_2 z}\right),$$

where $r_{1,2}=\frac{-\mu_1 \pm \sqrt{\mu_1^2+2q^2}}{2}$ — roots of the characteristic equation $r^2+\mu_1 r-0.5q^2=0$. These solutions can be overwritten in form

$$V_0(z)=E_0\sqrt{\frac{\mu_1^2+2q^2\cos^2\left(0.5\sqrt{\mu_1^2+2q^2}z\right)}{\mu_1^2+2q^2}}\times$$
$$\times\exp\left(-j\frac{\mu_1 z}{2}+j\varphi_0(z)\right)$$
$$V_1(z)=E_0\frac{q\sin\left(0.5\sqrt{\mu_1^2+2q^2}z\right)}{\sqrt{\mu_1^2+2q^2}}\exp\left(-j\frac{\mu_1 z}{2}+j\frac{\pi}{2}\right).$$

For phase $\varphi_0(z)$ following expression is true $tg\varphi_0=\frac{\mu_1}{\sqrt{\mu_1^2+2q^2}}tg\left(0.5\sqrt{\mu_1^2+2q^2}z\right)$. In this writing the amplitude and phase modulations of the transmitted and diffracted beams are separated. Ultrasonic (ultrasound) wave generates in medium amplitude-phase diffraction grating.

The intensities of the transmitted and diffracted waves going out of (withdrawal from) layer with width L of acousto-optic interaction equal

$$I_0(L)=V_0V_0^*=E_0^2\times$$
$$\times\left[1-\frac{2q^2}{\mu_1^2+2q^2}\sin^2\left(0.5\sqrt{\mu_1^2+2q^2}\cdot L\right)\right]$$
$$I_1(L)=I_{-1}(L)=V_1V_1^*= \tag{4}$$
$$=\frac{E_0^2 q^2}{\mu_1^2+2q^2}\sin^2\left(0.5\sqrt{\mu_1^2+2q^2}\cdot L\right)$$

Results of calculations and these discussions

The numerical calculations were made for diffraction of light beam wave length $\lambda_0=0.6328\,\mu m$ on propagating in water ultrasonic wave with sound wave length $\Lambda=150\,\mu m$. In this case system parameters are refractive index of medium $n_0=1.33$, value $\mu_1=\frac{\pi\lambda_0}{n_0\Lambda^2}=0.66$, parameter $q=\frac{2\pi\Delta n_0}{\lambda_0}=1.0\cdot10^5\cdot\Delta n_0$, wave parameter $Q=2\mu_1 L=1.32\cdot L$, the direction of diffraction maximums are defined by following angles $\sin\theta_m=m\cdot32\cdot10^{-4}\,rad=m\cdot11'$.

As is seen from obtained expressions (4) the behavior of the intensities owing to changes of range of interaction of light and sound L differ from the behavior of the intensities owing to changes power of sound wave, that described by parameter q. Only phase components of diffraction spectrum intensities explicitly depend on layer width of the acousto-optic interaction. Into these components wave parameter of diffraction $Q=2\mu_1 L$ enters also. On fig.1 the dimensionless intensities I_m/I_0 are plotted as a function of dimensionless Raman-Nath parameter qL. Here $I_0=E_0^2$ — intensity of incident radiation, I_m — intensity of diffraction maximums of zero and first orders. Two cases are considered. Firstly when width of sound layer is held fixed (L = const) and only sound intensity is changed. Secondly when sounds intensity is kept steadily, i.e. $\Delta n_0=const, q=const$, but only sound width L is varied. On these figures there are also the dependences light intensity in diffraction rays that was calculated in Raman-Nath approximation. It is seen that for values Raman-Nath parameter $qL\le1$ in both cases dependences for transmitted beam and first diffraction maximums coincide with distribution of Raman-Nath accordingly Bessell function. However, with increasing $qL>1$ the deviation of dependences from Bessell function are observed. At that these deviations are different for this two analyzing cases in the same region of changes of parameter qL.

The increase of the number of diffraction maximums brings to necessity solving system of linear homogeneous differential equations of first order with large number of equations. The results of numerical calculations for orders $m=0, 1, 2$ for just the same values of width of sound beam and index refractive that on fig.1 are shown on fig.2. It is seen, that in case of more accurate calculation with taking account of $m=0, 1, 2$ the coincidence with Raman-Nath approximation of first two maximums $m=0$ (transmitted radiation) and $m=1$ (first order) is more exact (better) than in case of calculations with account only $m=0,1$ over the same range of changes values of parameter (product) qL. And that with variation of sound beam width the difference from Bessell distribution is much (rather) more than at change intensity of sound wave when width of layer of acousto-optic interaction is fixed.

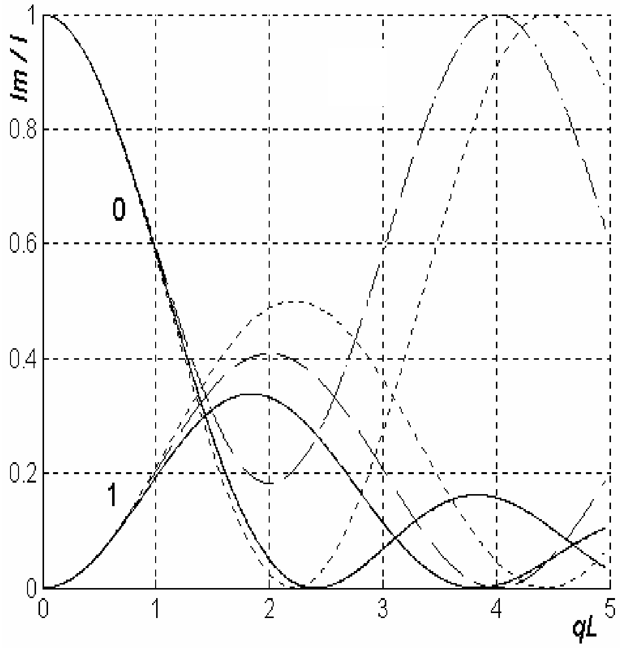

Fig. 1. Dependences of intensities of zero (0) and first (1) diffraction maximums on parameter qL, calculated by formula (4). Dashed lines– L=0.1 cm, Δn=(0 – 5·10^{-4}). Dotted lines – Δn=10^{-5}, L=(0 – 5) cm. Solid lines – Raman-Nath approximation

Numerical calculations was done with account of diffraction maximums $m=0, 1, 2, 3$ and showed that the values of amplitudes of diffraction maximums which was derived with using of lowest degree of approximations only these are defined more exactly with using of more high degrees of approximations.

The dependencies of the difference $\Delta I_m = \left(I_m^{(2)} - I_m^{(3)}\right)/I_0$ ($I_m^{(2)}$ — light intensity in m diffraction maximum calculated by approximation m = 0,1,2 and $I_m^{(3)}$ — light intensity in m diffraction maximum calculated in next approximation m = 0,1,2,3) are presented in fig. 3 and 4. It is seen that in one and the same range of values of dimensionless parameter qL the dependences of intensity of zero, first, second orders on sound layer size do not differ for given approximations. However for dependences on parameter q there is considerable difference especially for thin layers.

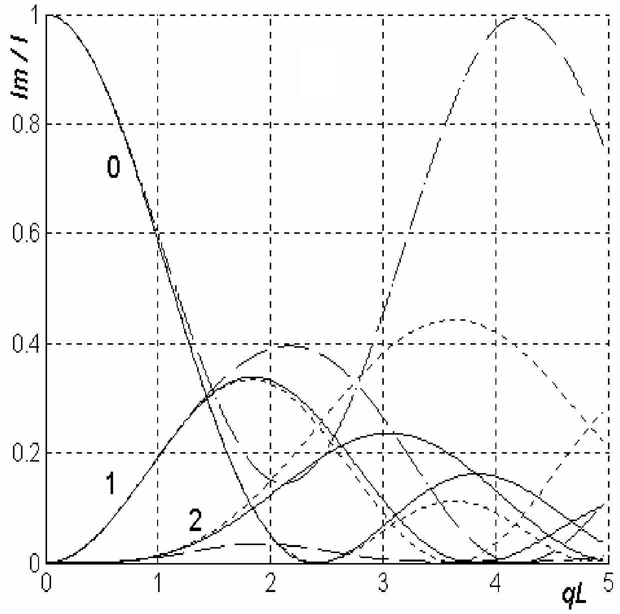

Fig. 2. Dependences of intensities of zero (0), first (1) and second (2) diffraction maximums on parameter qL. Dashed lines– L=0.1 cm, Δn=(0 – 5·10^{-4}). Dotted lines – Δn=10^{-5}, L=(0 – 5) cm. Solid lines – Raman-Nath approximat ion

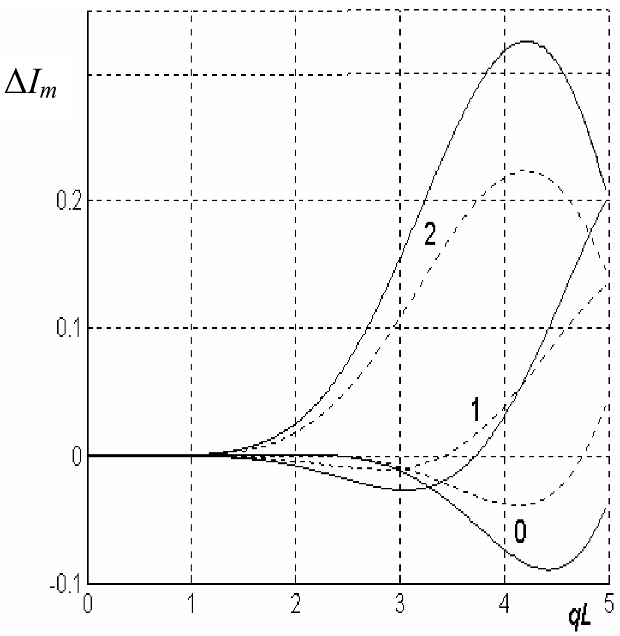

Fig. 3. Dependences of difference between solutions received by different approximations for zero (0), first (1) and second (2) diffraction maximums on value q (intensity of sound wave). Solid lines– L=0.1 cm, dashed lines –L=0.5 cm.

The dependences of the difference $\Delta I_m = I_m^{(3)}/I_0 - J_m^2$ on sound intensity with its constant width and on width of sound beam with constant its intensity are shown in fig.5 and 6. It is seen that behavior of diffraction spectra depending on intensity of sound wave for taken widths of interaction coincides with Raman-Nath approximation. However with increase of width sound beam even for small (not great) sound intensity the behavior of the intensity of diffraction maximums is distinguished from Bessell function.

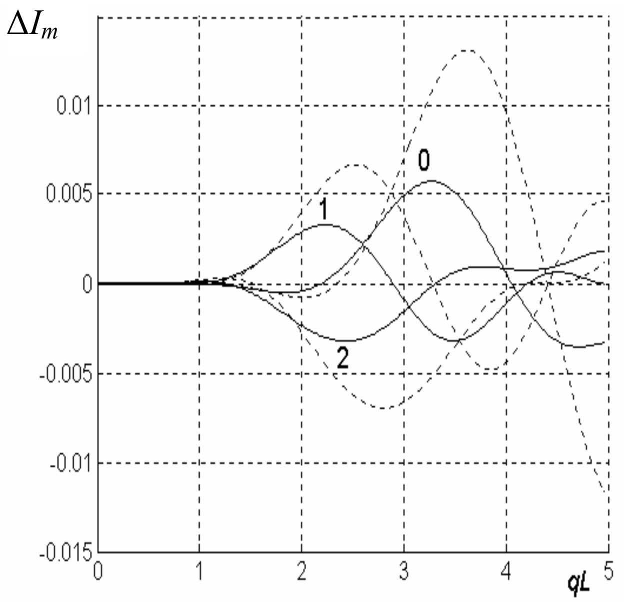

Fig. 4. Dependences of difference between solutions received by different approximations for zero (0), first (1) and second (2) diffraction maximums on width of sound layer L. Solid lines– $\Delta n=10^{-5}$, dashed lines – $\Delta n=1.2\cdot 10^{-5}$.

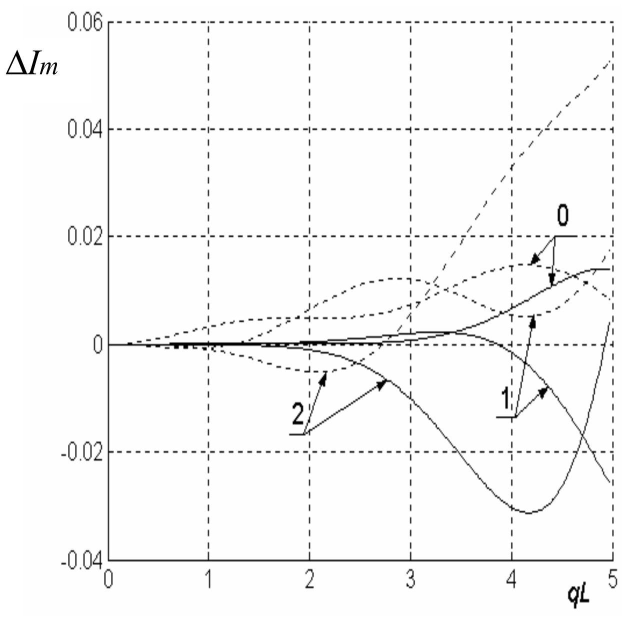

Fig. 5. Dependences of difference between Raman-Nath solution and solution received by approximation m=0,1,2,3 for zero (0), first (1) and second (2) diffraction maximums on value q (intensity of sound wave). Solid lines– L=0.1 cm, dashed lines –L=0.5 cm.

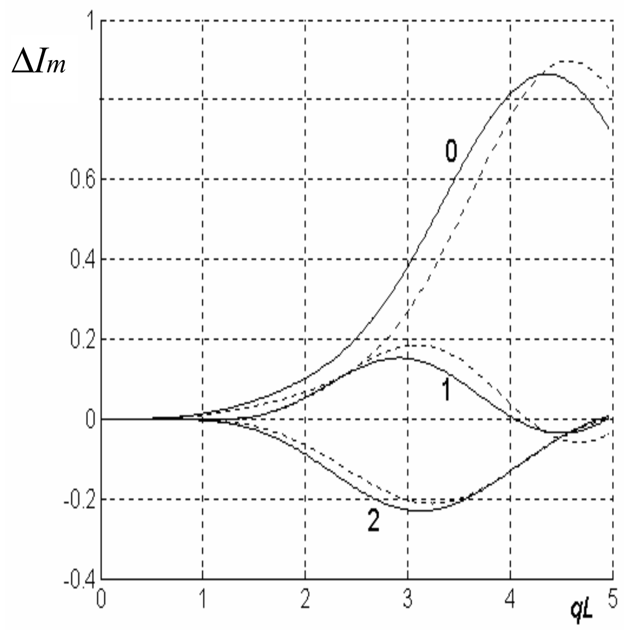

Fig. 6. Dependences of difference between Raman-Nath solution and solution received by approximation m=0,1,2,3 for zero (0), first (1) and second (2) diffraction maximums on width of sound layer L. Solid lines– $\Delta n=10^{-5}$, dashed lines – $\Delta n=1.2\cdot 10^{-5}$.

In present paper modifications of diffraction spectra with charge of sound beam width are investigated. The calculating dependences of light intensity in diffraction maximums zero (transmitted without angular deflection), first, second and third orders on amplitude of index refraction (sound intensity) with increase of sound beam width are presented in fig.7,8. It is seen that with moderate width of acousto-optic interaction when wave parameter $Q \approx 1$ the light intensity distribution in diffraction maximums there is far from Bessell function. The increase of width L produces to growth of light intensity oscillations with increase of sound intensity . At that even for examining orthogonal alignment of the interacting fields with increase of width of sound field the decreases of amplitudes of diffraction maximums of second and third orders are observed. The calculating dependences of light intensity in diffraction maximums are presented in fig.8 for width of sound field L=0.6 см, Q=2.3206. It is seen that for some values of Δn_0 only transmitted beam $m = 0$ and two symmetrical diffraction maximums first order $m = 1$ и $m = -1$ are remained in diffraction spectrum. At that, light intensities in diffraction maximums of second order and, particularly, third in this range of changes Δn_0 are neglect small. Besides, for some value of amplitude of index refraction the intensity of the transmitted light ray coincides with intensity of two refractive and equal one third of intensity of incidence light.

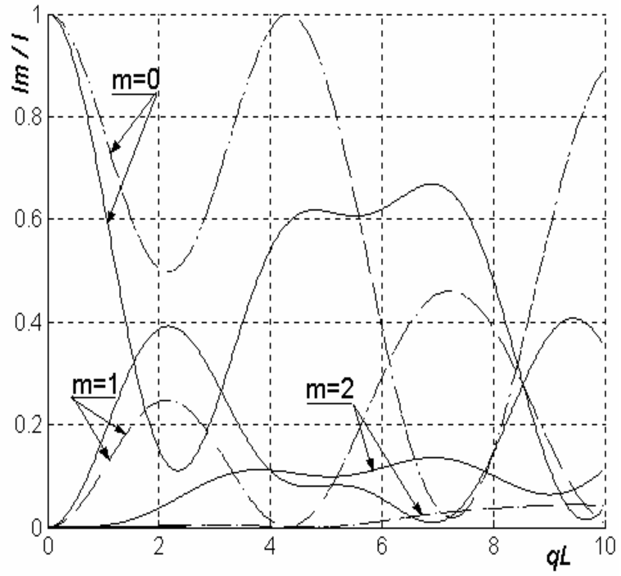

Fig.7. Computed dependences of light intensity in diffraction maximums on value q (intensity of sound wave). Solid lines – Q=2.66, L=2 cm, dashed lines – Q=5.32, L=4 cm.

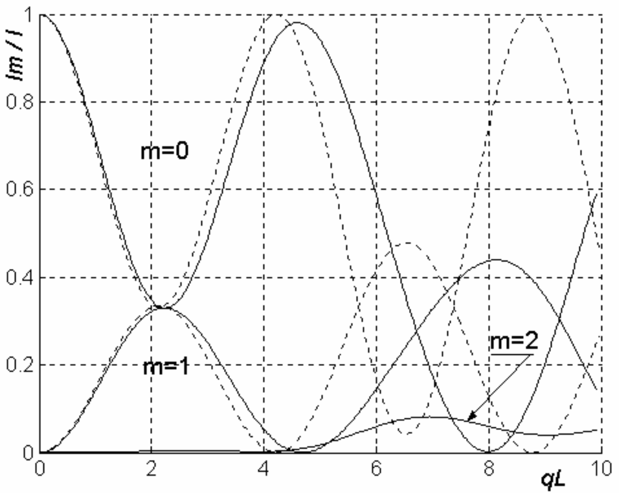

Fig. 8. Calculated dependences of light intensity in diffraction maximums on value q for different approximations. Solid lines– m=0,1,2,3; Q=4.26, L=3.21 cm, dot lines – m=0,1; Q=3.92, L=2.95 cm.

Conclusion

In present paper the theoretical analysis and calculation of the acousto-optic interaction was done for case of orthogonal orientation of interaction plane light and sound fields. At the same time dependences of diffraction spectra on both intensity of sound beam and its width are presented. The comparison of received results by numerical calculations with solutions received by Raman-Nath approximation was made. It was shown the behavior of diffraction spectra with changes of sound wave intensity substantially depend on sound beam width. Particularly, even with perpendicular incidence of light ray onto acoustic grating one can split light ray into three light ray with equal intensity propagated at small (not great) angles. In addition, investigation of behavior of diffraction spectra makes it possible to estimate intensity sound wave and connected with its parameters of medium.

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