A CHAOS-DYNAMICAL APPROACH TO ANALYSIS, PROCESSING AND FORECASTING MEASUREMENTS DATA OF THE CHAOTIC QUANTUM AND LASER SYSTEMS AND SENSORS

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Abstract. The paper is devoted to problem of development of new mathematical and computational tools for analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors). The chaos-geometric approach proposed includes a combined group of non-linear analysis and chaos theory methods such as the autocorrelation function method, multi-fractal formalism, wavelet analysis, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov’s exponents and Kolmogorov entropy analysis, surrogate data method, memory functions, neural networks algorithms. There are presented the most effective schemes for computing the Lyapunov’s exponents spectrum, Kaplan-Yorke dimension, Kolmogorov entropy etc.

Keywords: chaotic quantum systems and quantum sensors– analysis and processing the measurements data – chaos-geometric approach

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Анотація. Стаття присвячена проблемі розробки нових математичних і обчислювальних засобів для аналізу і обробки даних вимірювань хаотичних квантових і лазерних систем і квантових пристроїв (сенсорів). Пропонується хаос-геометричний підхід включає об’єднувану групу методів нелінійного аналізу та теорії хаосу, таких як метод автокореляційної функції, мультифрактальний формалізм, вейвлет-аналіз, метод взаємної інформації, метод кореляційного інтеграла, алгоритми помилкових найближчих сусідів і сурогатних даних, аналіз на основі показників Ляпунова і ентропії Колмогорова, формалізм функцій пам’яті, нейроморфежеві алгоритми і ін. Представлені найбільш ефективні схеми обчислення спектра показників Ляпунова, розмірності Каплана-Йорка, ентропії Колмогорова тощо.

Ключові слова: хаотичні квантові системи і квантові сенсори – аналіз і обробка даних вимірювань – хаос-геометричний підхід
1. Introduction

At present time a development of new mathematical and computational tools for analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors) is traditionally of a great importance and actuality for further development of modern quantum technologies, including quantum optics and spectroscopy, quantum and nano-and sensor electronics and different physical, chemical, even biological applications (see Refs. [1–12]). The last decades have seen an impressive progress in the understanding, modelling and even prediction of the evolutionary dynamics of different nonlinear complex systems and analysis and processing the corresponding measurements data.

For a long time different statistical methods such as autoregression, moving average or combined autoregression moving average (ARMA) methods and their refined generalizations have been used in numerical processing measurements data for different systems, however, in fact majority of these methods are linear and deal with known principal and computational difficulties [1]. Their nonlinear analogs such parametric or nonparametric ARMA type models have the known advantages and disadvantages. Both the accuracy and the reliability of analysis on the basis of these statistical methods could be strongly affected by the fundamental knowledge of the complex temporal structure and nonlinear interaction in a system.

In the last years a new approaches to environmental measurements data analysis and processing are provided by using methods of the non-linear analysis, chaos, dynamical systems theories [1-12]. In the modern technical studies a role of the correct measurement data (spatial and time series of main parameters) is very high.

Below we are interested by the measurement data for of chaotic quantum and laser systems and quantum devices (sensors) and their analysis and processing and development of new mathematical and computational tools for their correct analysis.

In this paper we consider a problem of development of new mathematical and computational tools for analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors). The chaos-geometric approach proposed includes a combined group of non-linear analysis and chaos theory methods such as the autocorrelation function method, multi-fractal formalism, wavelet analysis, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov’s exponents and Kolmogorov entropy analysis, surrogate data method, memory functions, neural networks algorithms. There are presented the most effective schemes for computing the Lyapunov’s exponents spectrum, Kaplan-Yorke dimension, Kolmogorov entropy etc. Their computational realization is based on the programs blocks of the “Geomath” and “Quantum Chaos” computational codes [13-23].

2. A Chaos-geometric approach to processing measurement data for complex systems

Our approach to analysis, processing and forecasting the measurement data of chaotic quantum and laser systems and quantum devices (sensors) is based on the fundamental results [1-6,13-15] and their generalizations. Formally, one could consider scalar chaotic quantum or laser system measurement parameter (say, an output amplitude) $s$ and write it as:

$$s(n) = s(t_0 + n\Delta t) = s(n),$$

where $t_0$ is a start time, $\Delta t$ is time step, and $n$ is a number of measurements.

As the preliminary step of the data processing it is useful to check the known Gottwald-Melbourne chaotic test [8]. It supposes studying the parameter $K$, which is determined by a limiting behavior of a root-mean-square shift:

$$M(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} [s(j+n) - s(j)]^2,$$

$$s(n) = \sum_{j=1}^{n} s(j) \cos(jc).$$

The cases of $K = 0$ and $K = 1$ correspond to a regular and chaotic dynamics respectively.
The first fundamental step of modelling is in reconstruction of the phase space using as well as possible information contained in \( s(n) \). From the mathematical viewpoint, this procedure results in set of \( d \)-dimensional vectors \( y(n) \) replacing scalar measurements. One should further operate with lagged variables \( s(n+\tau) \), where \( \tau \) is some integer to be defined, results in a coordinate system where a structure of orbits in phase space can be captured. Using a set of the time lags to create a vector in \( d \) dimensions,

\[
y(n) = [s(n), s(n + \tau), s(n + 2\tau), \ldots, s(n + (d-1)\tau)],
\]

(2)

the required coordinates are provided. The dimension \( d \) is defined as an embedding dimension, \( d_E \).

In Refs. [1,8,9] a few approaches to the choice of proper time lag are presented. This point is important for the subsequent reconstruction of phase space. The first approach is to compute the linear autocorrelation function \( C_\delta(\delta) \)

\[
C_\delta(\delta) = \frac{1}{N} \sum_{m=1}^{N} [s(m+\delta) - \bar{s}] [s(m) - \bar{s}]
\]

(3)

and to look for that time lag where \( C_\delta(\delta) \) first passes through zero. This gives a good hint of choice for \( \tau \) at that \( s(n+j\tau) \) and \( s(n + (j + 1) \tau) \) are linearly independent. However, a linear independence of two variables does not mean that these variables are nonlinearly independent since a nonlinear relationship can differs from linear one. It is therefore preferably to utilize approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows.

Let there are two systems, \( A \) and \( B \), with measurements \( a_i \) and \( b_k \). The amount one learns in bits about a measurement of \( a_i \) from measurement of \( b_k \) is given by arguments of information theory [10]

\[
I_{ab}(a_i, b_k) = \log_2 \left( \frac{P_{ab}(a_i, b_k)}{P(a_i)P(b_k)} \right),
\]

(4)

where the probability of observing \( a \) out of the set of all \( A \) is \( P(a) \), and the probability of finding \( b \) in a measurement \( B \) is \( P(b) \), and the joint probability of the measurement of \( a \) and \( b \) is \( P_{ab}(a_i, b_k) \). The mutual information \( I \) of two measurements \( a_i \) and \( b_k \) is symmetric and non-negative, and equals to zero if only the systems are independent.

The average mutual information between any value \( a_i \) from system \( A \) and \( b_k \) from \( B \) is the average over all possible measurements of \( I_{ab}(a_i, b_k) \),

\[
I_{ab}(\tau) = \sum_{a_i, b_k} P_{ab}(a_i, b_k) I_{ab}(a_i, b_k).
\]

(5)

To place this definition to a context of observations from a certain physical system, let us think of the sets of measurements \( s(n) \) as the \( A \), and of the measurements a time lag \( \tau \) later, \( s(n + \tau) \), as \( B \) set. The average mutual information between observations at \( n \) and \( n + \tau \) is then

\[
I_{ab}(\tau) = \sum_{a_i, b_k} P_{ab}(a_i, b_k) I_{ab}(a_i, b_k).
\]

(6)

Now we have to decide what property of \( I(\tau) \) we should select, in order to establish which among the various values of \( \tau \) we should use in making the data vectors \( y(n) \). One could remind that the autocorrelation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. The general redundancies detect all dependences in the time series, while the linear redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process resulting in the vibrations amplitude level variations can be concluded.

The fundamental goal of the \( d_E \) computing is in further reconstruction of the Euclidean space \( R^d \) large enough so that the set of points \( d_E \) can be unfolded without ambiguity. The embedding dimension, \( d_E \), must be greater, or at least equal, than a dimension of the corresponding chaotic attractor, \( d_A \), i.e. \( d_E \geq d_A \).
The correlation integral analysis is one of the widely used techniques to study the signatures attractor, resulting in the vibrations amplitude level variations can be concluded. Redundancies are sensitive only to linear structures. Further, a possible nonlinear nature of process linear redundancy and general redundancy, respectively, which was applied in the test for characterizing by an attractor, then the integral correlation dimension is defined by a limit of relation of the log $C(r)$ (C is a correlation integral) to log of the corresponding radius [11]:

$$C(r) = \lim_{N \to \infty} \frac{2}{N(N-1)} \sum_{1 \leq i < j \leq N} H(r - \| y_i - y_j \|),$$

(7)

where $H$ is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for $u \leq 0$, $r$ is the radius of sphere centered on $y_i$ or $y_j$, and $N$ is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius $r$ given by

$$d = \lim_{r \to 0} \frac{\log C(r)}{\log r},$$

(8)

where $d$ is correlation exponent that can be determined as the slope of line in the coordinates $\log C(r)$ versus $\log r$ by a least-squares fit of a straight line over a certain range of $r$, called the scaling region. In a chaotic case, the correlation exponent attains saturation with an increase in the embedding dimension. The saturation value of this exponent is defined as the correlation dimension ($d_L$) of the attractor.

Another approach to computing $d_L$ is the false nearest neighbour algorithm. As a rule, the simultaneous application of two methods provides more exact determination $d_L$. The nearest integer above the saturation value provides the minimum or optimum embedding dimension for reconstructing the phase-space or the number of variables necessary to model the dynamics of the system. This concept can be applied, since the embedding dimension determined by both the correlation dimension and false nearest neighbour algorithms are identical.

The further important step is determination of predictability, which can be estimated by the Kolmogorov entropy. The Kolmogorov entropy is proportional to a sum of the positive Lyapunov’s exponents. The Lyapunov’s exponents spectrum is one of the fundamental dynamical invariants for non-linear data system with a chaotic behavior. Since the Lyapunov’s exponents are defined as asymptotic average rates, they are independent of the initial conditions, and hence the choice of trajectory, and they do comprise an invariant measure of the attractor. An estimate of this measure is a sum of the positive Lyapunov’s exponents.

The estimate of the attractor dimension is provided by the Kaplan-Yorke conjecture $d_L$

$$d_L = j + \frac{\sum_{\alpha=1}^{j} \lambda_{\alpha}}{|\lambda_{j+1}|},$$

(9)

where $j$ is such that $\sum_{\alpha=1}^{j} \lambda_{\alpha} > 0$ and $\sum_{\alpha=1}^{j} \lambda_{\alpha} < 0$, and Lyapunov’s exponents are taken in descending order.

The dimension $d_L$ gives values close to the dimension estimates discussed earlier and is preferable when estimating high dimensions. To compute the Lyapunov’s exponents spectrum, we use a method with higher order polynomials fitted map [1].

In Table 1 we present the main blocks of an universal approach to analysis and processing measurement data for studied systems [1-5,13-23].

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tr>
<td>Preliminary study of data and assessment of</td>
<td>of the presence of chaos:</td>
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<td>the presence of chaos:</td>
<td>1. Test by Gottwald-Melbourne: $K \to 1$ – chaos;</td>
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<td></td>
<td>2. Fourier decompositions, irregular nature of change – chaos;</td>
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<td>spectrum of power...</td>
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Table 1. Chaos-dynamical approach to analysis and processing environmental measurement data for chaotic quantum and laser systems and devices (sensors)
II. The geometry of the phase space. Fractal Geometry:
4. Computing time delay $\tau$ (autocorrelation function or mutual information;
5. Computing embedding dimension $d_e$ by the method of correlation dimension or FNN algorithm;
6. Computing multi-fractal spectra. Wavelet analysis;

III. Prediction:
7. Computing global LEs: $\lambda_n$; Kaplan-Yorke dimension $d_k$, KE, average predictability measure
8. Determining the number of nearest neighbour points NN for the best prediction results;
9. Methods of nonlinear prediction, neural networks and quantum neural networks algorithms. Algorithm optimized trajectories, ...

3. Prediction model and conclusions

The most complex topic of a chaos-geometric approach is realization of correct prediction of a measurement data chaotic dynamics for studied systems and devices. We propose to use a new method, which is based on using the traditional concept of a compact geometric attractor, in which evolves the measurement data, plus the implementation of neural networks (NNN) algorithm [1,13,14]. The meaning of the concept is in the doctrine of evolution attractor in the phase space of the system and in a sense the simulation (“guessing”) temporal evolution. It’s about the fact that the phase space of a system orbit some continuously rolled on itself as a result of dissipative forces and the nonlinear part of the dynamics, so it is possible to find in the neighborhood of any point of the orbit $y(n)$ other points of the orbit $y'(n)$, $r = 1, 2, \ldots, N_y$ arriving in the neighborhood of $y(n)$ in different time moments which differ of $n$. Of course, then one can try to build different types of interpolation functions that take into account the whole neighborhood of the phase space, while explaining how the neighborhood evolve from $y(n)$ around all points set near $y(n+1)$. In terms of the theory of neural networks, the simulation of the evolution of the system can be described by some generalized evolutionary neural dynamic equations. Simulating further the evolution of complex systems as appropriate neural network evolution with elements of self-learning, self-adaptability, etc., there is a significant opportunity to improve the quality of prediction of the evolutionary dynamics of modelling the attractor in a chaotic system. Modelling attractor by some record in memory, neural system evolutionary process, i.e. the transition from the initial state to the (next) final state, can be represented by a model of reconstruction of the full record on distorted information, that is a model of associative recognition. Domain of attraction of different attractors are separated by separatrices or by certain surfaces in the phase space, structure of which is quite complex. However, it imitates the properties of the chaotic object. The next step is to construct a parameterized nonlinear function $F(x, a)$, which transform $y(n)$ to $y(n) \equiv y(n+1) = F(y(n), a)$, and use different, including the neural network criteria for determining the parameters $a$. As the functional form of displaying, one may use, for example, polynomial basis functions. A measure of the quality of the curve fit to the data, which is determined from the condition, how exactly coincide $y(k+1)$ with $F(y(k), a)$ is a local deterministic error: $e_r(k) = y(k+1) - F(y(k), a)$. If the mapping $F(y, a)$ is local, then for each neighbor to $y(k)$ point, $y^{(r)}(k)$ ($r = 1, 2, \ldots, N_y$) can be written as $e_D^{(r)}(k) = y(r, k + 1) - F(y^{(r)}(k), a)$, where $y(r, k + 1)$ is the point in phase space, which evolves $y(r, k)$. To measure the quality of the curve fit to the data, the local cost function has the form (in fact, the function value for the error): $W(\varepsilon, k) = \sum_{i=1}^{N_y} [e_D^{(r)}(k)]^2/\sum_{i=1}^{N_y} [y(k) - y(r, k)]^2$ and the parameters, determined by minimizing $W(\varepsilon, k)$, are dependent on parameter $a$. More formally, it is possible to start neural network algorithm, especially in terms of training an equivalent system of neural networks with the reconstruction and forecasting neural system state (correspondingly, correction of $a$).
Therefore, we presented the possible complex of mathematical and computational tools for analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors). The chaos-geometric approach proposed includes a combined group of non-linear analysis and chaos theory methods such as the autocorrelation function method, multi-fractal formalism, wavelet analysis, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov’s exponents and Kolmogorov entropy analysis, surrogate data method, memory functions, neural networks algorithms. It is self-understood that their concrete application will have some special peculiarities in dependence upon the measurement data quality of studied system or device.

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Summary

The aim of the work is to develop and present a new effective approach to analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors),
which are of a great importance for different applications in quantum optics and atomic spectroscopy, quantum and nano-and sensor electronics etc.

In the paper we consider new mathematical and computational tools for analysis and processing the measurements data of chaotic quantum and laser systems and quantum devices (sensors). The chaos-geometric approach proposed includes a combined group of non-linear analysis and chaos theory methods such as the autocorrelation function method, multi-fractal formalism, wavelet analysis, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov’s exponents and Kolmogorov entropy analysis, surrogate data method, memory functions, neural networks algorithms. There are presented the most effective schemes for computing the Lyapunov’s exponents spectrum, Kaplan-Yorke dimension, Kolmogorov entropy etc.

**Keywords:** chaotic quantum systems and quantum sensors – analysis and processing the measurements data – chaos-geometric approach

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**HAOS-DINAMICHIHIY PІДХІД DO АНАLІЗУ, ОБРОБКИ ТА ПРОГНОЗУВАННЯ ДАНИХ ВИМІРЮВАНЬ ДЛЯ ХАОТИЧНИХ КВАНТОВИХ СИСТЕМ І СЕНСОРІВ**

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**Реферат**

Метою роботи є розробка та представлення нового ефективного підходу до аналізу та обробки даних вимірювань для хаотичних квантових і лазерних систем та квантових приладів (датчиків), які мають велике значення для різних застосувань у квантовій оптиці та атомній спектроскопії, квантовій, нано-і сенсорній електроніці тощо.

Стаття присвячена проблемі розробки нових математичних і обчислювальних засобів для аналізу і обробки даних вимірювань хаотичних квантових і лазерних систем і квантових пристроїв (сенсорів). Пропонуємий хаос-геометричний підхід включає об’єднану групу методів нелінійного аналізу та теорії хаосу, таких як метод автокореляційної функції, мультифрактальний формалізм, вейвлет-аналіз, метод взаємної інформації, метод кореляційного інтеграла, алгоритми помилкових найближчих сусідів і сурогатних даних, аналіз на основі показників Ляпунова і ентропії Колмогорова, формалізм функцій пам’яті, нейромеморизовані алгоритми і ін. Представлені найбільш ефективні схеми обчислення спектра показників Ляпунова, розмірності Каплана-Йорка, ентропії Колмогорова тощо.

**Ключові слова:** хаотичні квантові системи і квантові сенсори – аналіз і обробка даних вимірювань – хаос-геометричний підхід