ПРОЕКТУВАННЯ І МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ СЕНСОРІВ

SENSORS DESIGN AND MATHEMATICAL MODELING

CHAOTIC DYNAMICS OF THE SEMICONDUCTOR GaAs / GaAlAs LASER WITHIN NONLINEAR CHAOS-GEOMETRIC INFORMATION ANALYSIS

G. P. Prepelitsa

Odessa State Environmental University, 15, Lvovskaya str., Odessa, Ukraine
e-mail: quantpre@mail.ru

Abstract. Using universal chaos-geometric and multisystem approach it is studied chaotic dynamics of the nonlinear processes in low- and high dimensional dynamics of a chaos generation in the semiconductor GaAs / GaAlAs laser device with retarded feedback. In order to make modelling chaotic dynamics it has been constructed improved complex system (with chaos-geometric, neural-network, forecasting, etc. blocks) that includes a set of new quantum-dynamic models and partially improved non-linear analysis methods including correlation (dimension D) integral, fractal analysis, average mutual information, false nearest neighbours, Lyapunov exponents (LE), Kolmogorov entropy (KE), power spectrum, surrogate data, nonlinear prediction, predicted trajectories, neural network methods etc. There are theoretically studied scenarios of generating chaos, obtained complete quantitative data on the characteristics of chaotic dynamics and topological and dynamic invariants, including Lyapunov exponents, Kolmogorov entropy, the limit of predictability and others.

Keywords: chaotic dynamics, semiconductor GaAs / GaAlAs laser, chaos-geometric approach

ХАОТИЧНА ДИНАМІКА НАПІВПРОВІДНИКОВОГО GaAs / GaAlAs ЛАЗЕРА: НЕЛІНІЙНИЙ ХАОС-ГЕОМЕТРИЧНИЙ ІНФОРМАЦІЙНИЙ АНАЛІЗ

Г. П. Препелиця

Анотація. На основі універсального хаос-геометричного і мультисистемного підходу вивчається низько та високо- розмірна динаміка генерації хаосу в напівпровідниковому GaAs / GaAlAs лазерному пристрої з запізненим зворотнім зв'язком. Для того, щоб виконати ефективне моделювання хаотичної динаміки нелінійних процесів в напівпровідникових системах та приладах розроблено комп'ютерний комплекс (з хаос- геометричним, нейромережевим блоками, блоком прогнозування, і т.і.), який, зокрема, включає в себе низку нових квантово-динамічних моделей динаміки процесів і покращені або принципово нові процедури та алгоритми нелінійного аналізу такі як метод ефективного кореляційного інтегралу, фрактальний аналіз, алгоритми середньої взаємної інформації та хибних найближчих сусідів, підхід до аналізу на основі показників Ляпунова, ентропія Колмогорова, метод сурогатних даних, моделі нелінійного прогнозу, спектральні методи, нейромережеві алгоритми тощо. Теоретично виявлені сценарії генерації хаосу, отримані кількісні дані про характеристики хаотичної динаміки, топологічні і динамічні інваріанти, зокрема, показники Ляпунова, ентропія Колмогорова, межа передбачуваності та інші.

Ключові слова: хаотична динаміка, напівпровідниковий GaAs / GaAlAs лазер, хаос-геометричний підхід

ХАОТИЧЕСКАЯ ДИНАМИКА ПОЛУПРОВОДНИКОВОГО GaAs / GaAlAs ЛАЗЕРА: НЕЛИНЕЙНЫЙ ХАОС-ГЕОМЕТРИЧЕСКИЙ ИНФОРМАЦИОННЫЙ АНАЛИЗ

Г. П. Препелица

Аннотация. На основе универсального хаос-геометрического и мультисистемного подхода изучается низко и высоко-размерная динамика генерации хаоса в полупроводниковом GaAs / GaAlAs лазерном устройстве с запаздывающей обратной связью.

Для того, чтобы выполнить эффективное моделирование хаотической динамики нелинейных процессов в полупроводниковых системах и приборах разработан компьютерный комплекс (с хаос- геометрическим, нейросетевым блоками, блоком прогнозирования, и т.д.), который, в частности, включает в себя ряд новых квантово -динамических моделей динамики процессов и улучшенные или принципиально новые процедуры и алгоритмы нелинейного анализа такие как метод эффективного корреляционного интеграла, фрактальный анализ, алгоритмы средней взаимной информации и ложных ближайших соседей, подход к анализу на основе показателей Ляпунова, энтропия Колмогорова, метод суррогатных данных, модели нелинейного прогноза, спектральные методы, нейросетевые алгоритмы и т.д. Теоретически выявлены сценарии генерации хаоса, получены количественные данные о характеристиках хаотической динамики, топологические и динамические инварианты, в частности, показатели Ляпунова, энтропия Колмогорова, предел предсказуемости и другие.

Ключевые слова: хаотическая динамика, полупроводниковый GaAs / GaAlAs лазер, хаос-геометрический подход

1. Introduction

In a modern quantum electronics and laser physics etc there are many systems and devices (such as multi-element semiconductors and gas lasers etc), dynamics of which can exhibit chaotic behaviour. These systems can be considered in the first approximation as a grid of autogenerators (quantum generators), coupled by different way [1-9].

In [6,7] we presented an application of a new and advanced known non-linear analysis, chaos theory and information technology methods [6-14] to studying non-linear dynamics of the erbium one-ring fibre laser (EDFL, 20.9mV strength, $\lambda = 1550.190$nm) with the control parameters: the modulation frequency f and dc bias voltage of the electro-optical modulator. Technique of non-linear analysis includes a whole sets of new algorithms and advanced known methods such as the wavelet analysis, multi-fractal formalism, mutual information approach, correlation integral analysis, false nearest neighbour algorithm, Lyapunov exponent's (LE) analysis, and surrogate data method, neural networks prediction approach etc (see details in Refs. [6-14]).

In this paper we present the results of the first full quantitative study of low- and high dimensional dynamics of a chaos generation in the semiconductor GaAs / GaAlAs laser device with retarded feedback within a new non-linear analysis, chaos theory and information technology approach [6-11].

2. Methods of non-linear analysis and a chaos theory

As used non-linear analysis, chaos theory and information technology methods to studying non-linear dynamics of the laser systems have been earlier in details presented [6-14] here we are limited only by the key ideas. As usually, we formally consider scalar measurements $s(n) = s(t_0 + n\Delta t) = s(n)$, where t_0 is the start time, Δt is the time step, and is n the number of the measurements. Packard et al. [15] introduced the method of using time-delay coordinates to reconstruct the phase space of an observed dynamical system. The direct use of the lagged variables $s(n + \tau)$, where t is some integer to be determined, results in a coordinate system in which the structure of orbits in phase space can be captured. First

approach to compute τ is based on the linear auto-correlation function. The second method is an approach with a nonlinear concept of independence, e.g. the average mutual information. Briefly, the concept of mutual information can be described as follows [5,7]. One could remind that the auto-correlation function and average mutual information can be considered as analogues of the linear redundancy and general redundancy, respectively, which was applied in the test for nonlinearity. If a time series under consideration have an n-dimensional Gaussian distribution, these statistics are theoretically equivalent.

The goal of the embedding dimension determination is to reconstruct a Euclidean space R^d large enough so that the set of points d_A can be unfolded without ambiguity. There are several standard approaches to reconstruct the attractor dimension (see, e.g., [11-17]), but let us consider in this study two methods only. The correlation integral analysis is one of the widely used techniques to investigate the signatures of chaos in a time series. The analysis uses the correlation integral, $C(r)$, to distinguish between chaotic and stochastic systems. To compute the correlation integral, the algorithm of Grassberger and Procaccia [17] is the most commonly used approach. According to this algorithm, the correlation integral is

$$C(r) = \lim_{N \to \infty} \frac{2}{N(n-1)} \sum_{\substack{i,j \\ (1 \le i < j \le N)}} H\left(r - \parallel \mathbf{y}_i - \mathbf{y}_j \parallel\right) \quad (1)$$

where H is the Heaviside step function with $H(u) = 1$ for $u > 0$ and $H(u) = 0$ for $u \le 0$, r is the radius of sphere centered on $\mathbf{y}_i$ or $\mathbf{y}_j$, and N is the number of data measurements. If the time series is characterized by an attractor, then the integral $C(r)$ is related to the radius r given by

$$d = \lim_{\substack{r \to 0 \\ N \to \infty}} \frac{\log C(r)}{\log r}, \quad (2)$$

where d is correlation exponent that can be determined as the slop of line in the coordinates $\log C(r)$ versus $\log r$ by a least-squares fit of a straight line over a certain range of r, called the scaling region.

If the correlation exponent attains saturation with an increase in the embedding dimension, then the system is generally considered to ex-

hibit chaotic dynamics. The saturation value of the correlation exponent is defined as the correlation dimension (d_2) of the attractor. The nearest integer above the saturation value provides the minimum or optimum embedding dimension for reconstructing the phase-space or the number of variables necessary to model the dynamics of the system. On the other hand, if the correlation exponent increases without bound with increase in the embedding dimension, the system under investigation is generally considered stochastic. There are certain important limitations in the use of the correlation integral analysis in the search for chaos. For instance, the selection of inappropriate values for the parameters involved in the method may result in an underestimation (or overestimation) of the attractor dimension. Consequently, finite and low correlation dimensions could be observed even for a stochastic process. To verify the results obtained by the correlation integral analysis, we use surrogate data method.

The method of surrogate data is an approach that makes use of the substitute data generated in accordance to the probabilistic structure underlying the original data [6-8]. This means that the surrogate data possess some of the properties, such as the mean, the standard deviation, the cumulative distribution function, the power spectrum, etc., but are otherwise postulated as random, generated according to a specific null hypothesis. Here, the null hypothesis consists of a candidate linear process, and the goal is to reject the hypothesis that the original data have come from a linear stochastic process. One reasonable statistics suggested by Theiler et al. (look [6]) is obtained as follows. If we denote Q_{orig} as the statistic computed for the original time series and Q_{si} for ith surrogate series generated under the null hypothesis and let μ_s and σ_s denote, respectively, the mean and standard deviation of the distribution of Q_s, then the measure of significance S is given by

$$S = \frac{|Q_{orig} - \mu_s|}{\sigma_s}. \qquad (3)$$

An S value of ~2 cannot be considered very significant, whereas an S value of ~10 is highly significant. To detect nonlinearity in the amplitude level data, the one hundred realizations of surrogate data sets were generated according to

a null hypothesis in accordance to the probabilistic structure underlying the original data. Often, a significant difference in the estimates of the correlation exponents, between the original and surrogate data sets, can be observed. In the case of the original data, a saturation of the correlation exponent is observed after a certain embedding dimension value (i.e., 6), whereas the correlation exponents computed for the surrogate data sets continue increasing with the increasing embedding dimension. The high significance values of the statistic indicate that the null hypothesis (the data arise from a linear stochastic process) can be rejected and hence the original data might have come from a nonlinear process. It is worth consider another method for determining d_E that comes from asking the basic question addressed in the embedding theorem: when has one eliminated false crossing of the orbit with itself which arose by virtue of having projected the attractor into a too low dimensional space? By examining this question in dimension one, then dimension two, etc. until there are no incorrect or false neighbours remaining, one should be able to establish, from geometrical consideration alone, a value for the necessary embedding dimension. Such an approach was originally described by Kennel et al. [16].

The LE are the dynamical invariants of the nonlinear system. In a general case, the orbits of chaotic attractors are unpredictable, but there is the limited predictability of chaotic physical system, which is defined by the global and local LE [6-9,17-21]. A negative exponent indicates a local average rate of contraction while a positive value indicates a local average rate of expansion. In the chaos theory, the spectrum of LE is considered a measure of the effect of perturbing the initial conditions of a dynamical system. Note that both positive and negative LE can coexist in a dissipative system, which is then chaotic. Since the LE are defined as asymptotic average rates, they are independent of the initial conditions, and therefore they do comprise an invariant measure of attractor. In fact, if one manages to derive the whole spectrum of the LE, other invariants of the system, i.e. Kolmogorov entropy and attractor's dimension can be found. The Kolmogorov entropy, K, measures the average rate at which information about the state is lost with time. An estimate

of this measure is the sum of the positive LE. The inverse of the Kolmogorov entropy is equal to an average predictability. Estimate of dimension of the attractor is provided by the Kaplan and Yorke conjecture [20]:

$$d_L = j + \frac{\sum_{\alpha=1}^{j} \lambda_\alpha}{|\lambda_{j+1}|} , \qquad (4)$$

where j is such that $\sum_{\alpha=1}^{j} \lambda_\alpha > 0$ and $\sum_{\alpha=1}^{j+1} \lambda_\alpha < 0$, and the LE λ_α are taken in descending order. There are a few approaches to computing the LE. One of them computes the whole spectrum and is based on the Jacobi matrix of system [6]. In the case where only observations are given and the system function is unknown, the matrix has to be estimated from the data. In this case, all the suggested methods approximate the matrix by fitting a local map to a sufficient number of nearby points. In our work we use the method with the linear fitted map proposed by Sano and Sawada [21], although the maps with higher order polynomials can be also used. To calculate the spectrum of the LE from the amplitude level data, one could determine the time delay t and embed the data in the four-dimensional space. In this point it is very important to determine the Kaplan-Yorke dimension and compare it with the correlation dimension, defined by the Grassberger-Procaccia algorithm. The estimations of the Kolmogorov entropy and average predictability can further show a limit, up to which the amplitude level data can be on average predicted.

3. The results of chaos generation analysis in the semiconductor GaAs / GaAlAs laser device with retarded feedback

Fischer et al [5] have carried out the experimental studying dynamics of a chaos generation in the semiconductor GaAs / GaAlAs Hitachi HLP1400 laser; an instability is generated by means of the retarded feedback during changing the control parameter such as the feedback strength m (or in fact an injection current). Of course, depending on the system m there is appeared a multi-stability of different states with the modulation period: $T_n = 2\tau/(2n+1)$, n=0, 1,2,… The state of n = 0 is called as a ground one. With respect to the frequency modulation, other states are called as the third harmonic, fifth harmonic and so on.

In the figure 1 we list the measured data on the time-dependent intensities for a semiconductor laser device with feedback: a) – the time series, which illustrates a chaotic wandering between the ground state and the state of the third harmonic; b) the time series for a system in a state of the global chaotic attractor.
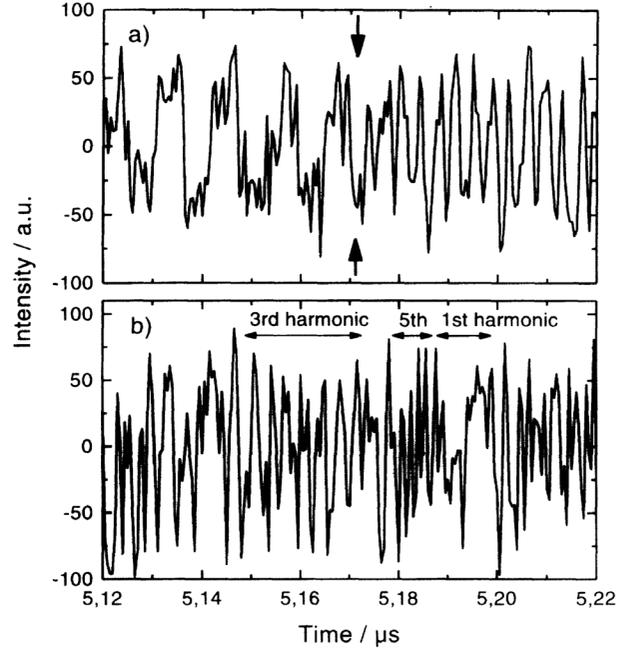

Figure 1. The time series of intensity in the GaAs/ GaAlAs Hitachi HLP1400 laser (the measured data).

In the Table 1 we present our original data on the correlation dimension d_2, the embedding dimension, computed on the basis of the false nearest neighboring points algorithm (d_N) with percentage of false neighbors (%) which are calculated for different lag times t . The data are listed for two regimes: I. chaos and II hyperchaos. In Table 2 we present our original data on the Lyapunov's exponents (LE), Kaplan-Yorke attractor dimensions, the Kolmogorov entropy K_{entr}. One can see that there are the LE positive and negative values

Table 1.

The correlation dimension d_2, the embedding dimension, computed on the basis of the false nearest neighboring points algorithm (d_N) with percentage of false neighbors (%) which are calculated for different lag times τ

Chaos regime (I)			Hyperchaos regime (II)		
τ	d_2	(d_N)	τ	d_2	(d_N)
58	3.4	5 (8.1)	67	8.4	11 (15)
6	2.2	4 (1.05)	10	7.4	8 (3.4)
8	2.2	4 (1.05)	12	7.4	8 (3.4)

Таблиця 2.

the Lyapunov's exponents (LE): λ_1-λ_4, the Kaplan-Yorke attractor dimension d_L and the Kolmogorov entropy K_{entr}

Regime	λ_1	λ_2	λ_3	λ_4	d_L	K_{entr}
Chaos (I)	0.151	0.00001	-0.188	-0.067	1.8	0.15
Hyperchaos (II)	0.517	0.192	-0.139	-0.042	7.1	0.71

The resulting Kaplan-Yorke dimensions in both cases are very similar to the correlation dimension, which is determined using the Grassberger-Procachia algorithm. The Kaplan-Yorke dimension is less than the embedding dimension that confirms the correct choice of the latter. A scenario of chaos generation is in converting initially periodic states into individual chaotic states with increasing the parameter m through a sequence of the period doubling bifurcations. Further there is appeared a global chaotic attractor after the merging individual chaotic attractors according a few complicated scenario.

References

1. Wiggins S. Introduction to applied nonlinear dynamical systems and chaos.-N.-Y: Springer, 1997.-688 p.
2. Kuznetsov A. P., Kuznetsov S. P., Ryskin N. M., Isaeva O. B., Non-linearity: From oscillations to chaos.-M: NIS RHD.-2006.-184p.
3. Ott E., Chaos in dynamical systems.-Cambridge: Cambridge Univ.Press: 2002.- 490p.
4. Gallager R. G., Information theory and reliable communication, Wiley, New York.-1986.
5. Fischer I., Hess O., Elsaber W., Go-bel E., High-dimensional chaotic dynamics of an external cavity semiconductor laser// Phys.Rev.Lett.-1994.-Vol.73.-P. 2188-2192.
6. Prepelitsa G. P., New nonlinear analysis, chaos theory and information technology approach to studying dynamics of the erbium one-ring fibre laser// Photoelectronics".-2015.-Vol.24.-P. 38-43.
7. Prepelitsa G. P., New chaos-geometric and information technology analysis of chaotic generation regime in a single-mode laser system with absorbing cell// International Scientific Journal «Proceedings of International Geometry Center».-2015.-Vol.8,N2.-P. 100-108.
8. Prepelitsa G. P., Glushkov A. V., Lepikh Ya. I., Buyadzhi V. V., Ternovsky V. B., Zaichko P. A., Chaotic dynamics of non-linear processes in atomic and molecular systems in electromagnetic field and semiconductor and fiber laser devices: New approaches, Uniformity and Charm of chaos// Sensors Electronics and Microsystems Technologies.-2014.-Vol.11,N4.-P.43-57.
9. Prepelitsa G. P., Nonlinear dynamics of quantum and laser systems with elements of a chaos//Photoelectronics.-2014.-Vol.23.-P. 96-106.
10. Glushkov A. V., Khetselius O. Yu., Prepelitsa G. P., Svinarenko A. A., Geometry of Chaos I: Theoretical basis's of a consistent combined approach // International Scientific Journal "Proceedings of International Geometry Center".-2013.-Vol.6,N1.-P.67-79.
11. Glushkov A. V., Prepelitsa G. P., Svinarenko A. A., Zaichko P. A. Studying interaction dynamics of the non-linear vibrational systems within non-linear prediction method (application to quantum autogenerators)// Dynamical Systems Theory, Eds. J. Awrejcewicz, M. Kazmierczak, P. Olejnik, J. Mrozowski (WSEAS, Lodz).-2013.-Vol.T1.-P.467-477.
12. Glushkov A. V., Svinarenko A. A.,

Buyadzhi V. V., Zaichko P. A., Ternovsky V. B., Chaos-geometric attractor and quantum neural networks approach to simulation chaotic evolutionary dynamics during perception process// Advances in Neural Networks, Fuzzy Systems and Artificial Intelligence, Series: Recent Advances in Computer Engineering, Ed. J.Balicki. (Gdansk, WSEAS Pub.).-2014.-Vol.21.-P.143-150.

13. Prepelitsa G. P., Buyadzhi V. V., Ternovsky V. B. Non-linear analysis of chaotic self-oscillations in backward-wave tube// Photoelectronics.-2013.-Vol.22.-P.103-107.

14. Glushkov A. V., Khokhlov V. N., Tsenenko I. A., Atmospheric teleconnection patterns and eddy kinetic energy content: wavelet analysis//Nonlinear Proc.in Geophys.-2004.-Vol.11.-P.285-293.

15. Packard N., Crutchfield J., Farmer J., Shaw R., Geometry from time series// Phys.Rev.Lett-1988.-Vol.45.-P.712–716.

16. Kennel M., Brown R., Abarbanel H., Determining embedding dimension for phase-space reconstruction using geometrical construction//Phys. Rev.A.-1992.-Vol.45.-P.3403–3411.

17. Grassberger P., Procaccia I., Measuring the strangeness of strange attractors// Physica D.-1983.-Vol.9.-P.189–208.

18. Takens F., Detecting strange attractors in turbulence//Dynamical systems and turbulence Ed. Rand D., Young L. – N.-Y.: Springer, 1981.-P.366–381.

19. Havstad J. W., Ehlers C. L. Attractor dimension of nonstationary dynamical systems from small data sets // Phys. Rev. A. – 1989. – Vol. 39. – P. 845-853.

20. Kaplan J. L., Yorke J. A. Chaotic behavior of multidimensional difference equations // Functional differential equations and approximations of fixed points. Lecture Notes in Mathematics No. 730 / H.-O. Peitgen, H.-O. Walter (Eds.). – Berlin: Springer, 1979. – P. 204-227.

21. Sano M., Sawada Y. Measurement of the Lyapunov spectrum from a chaotic time series // Phys. Rev. Lett. – 1985. – Vol. 55. – P. 1082-1085.

CHAOTIC DYNAMICS OF THE SEMICONDUCTOR GaAs / GaAlAs LASER WITHIN NONLINEAR CHAOS-GEOMETRIC INFORMATION ANALYSIS

G. P. Prepelitsa

Odessa State Environmental University, 15, Lvovskaya str., Odessa, Ukraine
e-mail: quantpre@mail.ru

Summary

Using universal chaos-geometric and multisystem approach it is studied chaotic dynamics of the nonlinear processes in low- and high dimensional dynamics of a chaos generation in the semiconductor GaAs / GaAlAs laser device with retarded feedback. In order to make modelling chaotic dynamics it has been constructed improved complex system (with chaos-geometric, neural-network, forecasting, etc. blocks) that includes a set of new quantum-dynamic models and partially improved nonlinear analysis methods including correlation (dimension D) integral, fractal analysis, method of the average mutual information, algorithm of the false nearest neighbours, Lyapunov exponents, scheme of computing the Kolmogorov entropy, analysis of the power spectrum, method of the surrogate data, method of nonlinear prediction (scheme of the predicted trajectories), neural network methods etc. There are theoretically studied scenarios of generating chaos, obtained complete quantitative data on the characteristics of chaotic dynamics and topological and dynamic invariants, including Lyapunov exponents, Kolmogorov entropy, the limit of predictability and others. It is shown that the firstly arising periodic states turns into individual chaotic states and then global chaotic attractor with scenario through period-doubling bifurcation, which then significantly modified. There are firstly presented original data on the Lyapunov exponents (+, +), correlation dimension (chaos – 2.2; hyperchaos – 7.4), embedding dimension , Kaplan-York dimension and Kolmogorov entropy.

Keywords: chaotic dynamics, semiconductor GaAs / GaAlAs laser, chaos-geometric approach

ХАОТИЧНА ДИНАМІКА НАПІВПРОВІДНИКОВОГО GaAs / GaAlAs ЛАЗЕРА: НЕЛІНІЙНИЙ ХАОС-ГЕОМЕТРИЧНИЙ ІНФОРМАЦІЙНИЙ АНАЛІЗ

Г. П. Препелиця

Одеський державний екологічний університет, вул. Львівська, 15, Одеса, 65016, Україна
e-mail: quantpre@mail.ru

Реферат

На основі універсального хаос-геометричного і мультисистемного підходу вивчається низько та високо- розмірна динаміка генерації хаосу в напівпровідниковому GaAs / GaAlAs лазерному пристрої з запізненим зворотнім зв'язком. Для того, щоб виконати ефективне моделювання хаотичної динаміки нелінійних процесів в напівпровідникових системах та приладах розроблено комп'ютерний комплекс (з хаос- геометричним, нейромережевим блоками, блоком прогнозування, і т.і.), який, зокрема, включає в себе низку нових квантово-динамічних моделей динаміки процесів і покращені або принципово нові процедури та алгоритми нелінійного аналізу такі як метод ефективного кореляційного інтегралу, фрактальний аналіз, алгоритми середньої взаємної інформації та хибних найближчих сусідів, підхід до аналізу на основі показників Ляпунова, ентропія Колмогорова, метод сурогатних даних, моделі нелінійного прогнозу, спектральні методи, нейромережеві алгоритми тощо. Теоретично виявлені сценарії генерації хаосу, отримані кількісні дані про характеристики хаотичної динаміки, топологічні і динамічні інваріанти, зокрема, показники Ляпунова, ентропія Колмогорова, межа передбачуваності та інші. Показано, що, по-перше, що виникаючі періодичні стани перетворюється в окремі хаотичні стани, а потім у глобальний атрактор за хаотичним сценарієм через біфуркації подвоєння періоду, який приймає ускладнену форму. Представлені розраховані чисельні значення щодо параметрів динаміки системи, зокрема, по показникам Ляпунова (+, +), кореляційній розмірності (у режимі хаосу - 2,2; у режимі гіперхаосу - 7.4), вкладеній розмірності, розмірності Каплан-Йорка, енергії Колмогорова тощо.

Ключові слова: хаотична динаміка, напівпровідниковий GaAs / GaAlAs лазер, хаос-геометричний підхід